Generalized Material-Independent PML Absorbers for the FDTD Simulation of Electromagnetic Waves in Arbitrary Anisotropic Dielectric and Magnetic Media

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***Abstract*—To simply and effectively absorb waves propagating in anisotropic materials consisting of both arbitrary permittivity and permeability tensors, generalized material-independent perfectly matched layer (GMIPML) absorbers are proposed. Within the GMIPML absorbers, electric displacement D and flux density B are directly absorbed, whereas electric field E and magnetic field H are simultaneously absorbed through the relations between E and D as well as H and B. The proposed GMIPML absorber is validated by analyzing two-dimensional (2-D) hybrid waves.**

***Index Terms*—Absorbing boundary conditions, anisotropic media, FDTD.**

I. Introduction

THE perfectly matched layer (PML) absorbing boundary condition (ABC) developed by Berenger [1] has been proven to be one of the best ABC's for the truncation of finite-difference time-domain (FDTD) grids. Recently, application of Berenger's PML to diagonal anisotropic media was investigated [2]–[4] with the approaches mainly based on $\mathbf{E}$ and $\mathbf{H}$ formulations. Particularly, inside the PML absorbers [2]–[4], the electric conductivity σ^E and magnetic conductivity σ^H are involved, which results in that $\mathbf{E}$ and $\mathbf{H}$ are (directly) absorbed by the PML absorbers. Although the PML absorbers have been successfully applied to *diagonal* anisotropic media [2]–[4], extension of them to *arbitrary* anisotropic cases (especially when both arbitrary permittivity $[\varepsilon]$ and permeability $[\mu]$ tensors exist) is rather difficult and cumbersome, and it was proven [5] that under certain circumstances no PML matching condition can be derived even for the simple case when only the arbitrary anisotropic dielectric is considered. This is due to the fact that the conductivities σ^E and σ^H used in the PML are *direct* functions of the tensors $[\varepsilon]$ and/or $[\mu]$. However, very recent investigations [6], [7] indicate that the above difficulties can be easily overcome if the material-independent PML absorbers are adopted. But the material-independent PML absorbers were separately developed for arbitrary anisotropic dielectric [6] and magnetic [7] media. In this letter, the ideas presented in [6] and [7] are further extended to materials consisting of both arbitrary permittivity and permeability tensors. In particular, in the proposed GMIPML absorbers $\mathbf{D}$ and $\mathbf{B}$ are directly absorbed, whereas $\mathbf{E}$ and $\mathbf{H}$ are concurrently absorbed. Because the conductivities σ^D and σ^B used in the proposed GMIPML are not functions of the tensors $[\varepsilon]$ and $[\mu]$ (as well as other material properties, such as loss and nonlinearity), thus, the absorber proposed in this letter has the property of material independence. Validation of the proposed GMIPML is carried out by analyzing two-dimensional (2-D) hybrid waves propagating in uniaxial anisotropic media.

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II. Theory

Let an anisotropic medium consisting of both arbitrary permittivity and permeability 3×3 tensors lie in the XY-plane. Hence, a hybrid wave (i.e., none of the six components of $\mathbf{E}$ and $\mathbf{H}$ is zero) will propagate if one of the off-diagonal elements $(\varepsilon_{xz}, \varepsilon_{yz}, \mu_{xz},$ or $\mu_{yz})$ of the tensors is not zero. If a variant of Yee's unit cell (i.e., the components of the $\mathbf{D}$ (or $\mathbf{B}$) field are located at the same positions as the components of the $\mathbf{E}$ (or $\mathbf{H}$) field) is adopted, then the hybrid wave in the GMIPML absorbers (note only D_z and B_z need to be split) is governed by

$$\frac{\partial D_x}{\partial t} + \sigma_y^D D_x = \frac{\partial H_z}{\partial y}, \quad \frac{\partial D_y}{\partial t} + \sigma_x^D D_y = -\frac{\partial H_z}{\partial x} \tag{1.1}$$

$$\frac{\partial D_{zx}}{\partial t} + \sigma_x^D D_{zx} = \frac{\partial H_y}{\partial x}, \quad \frac{\partial D_{zy}}{\partial t} + \sigma_y^D D_{zy} = -\frac{\partial H_x}{\partial y} \tag{1.2}$$

$$\frac{\partial B_x}{\partial t} + \sigma_y^B B_x = -\frac{\partial E_z}{\partial y}, \quad \frac{\partial B_y}{\partial t} + \sigma_x^B B_y = \frac{\partial E_z}{\partial x} \tag{1.3}$$

$$\frac{\partial B_{zx}}{\partial t} + \sigma_y^B B_{zx} = -\frac{\partial E_y}{\partial x}, \quad \frac{\partial B_{zy}}{\partial t} + \sigma_y^B B_{zy} = \frac{\partial E_x}{\partial y} \tag{1.4}$$

where the parameters $\sigma_x^D, \sigma_y^D, \sigma_x^B,$ and σ_y^B (which, respectively, relate to $\mathbf{D}$ and $\mathbf{B}$) are the conductivities used in the

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proposed GMIPML absorbers. The matching conditions are

$$\sigma_x^D = \sigma_x^B, \quad \sigma_y^D = \sigma_y^B. \tag{2}$$

It should be noticed that, unlike the electric conductivity $\sigma^E(\rho)$ [1], ε_0 is not involved in the expression of the theoretical reflection factor $R(\theta)$ for $\sigma^D(\rho)$. Especially, $R(\theta)$ used for $\sigma^D(\rho)$ in the GMIPML is

$$R(\theta) = e^{-2(\cos\theta/c)\int_0^\delta \sigma^D(\rho)\,d\rho}. \tag{3}$$

Besides, because within the GMIPML absorbers $\mathbf{D}$ and $\mathbf{B}$ (instead of $\mathbf{E}$ and $\mathbf{H}$) are directly absorbed as well as $\mathbf{E}$ and $\mathbf{H}$ are not split, updating equations used for $\mathbf{D}$ and $\mathbf{B}$ are slightly different from those used for $\mathbf{E}$ and $\mathbf{H}$ [1]. For example, updating equations for D_{zx} and B_{zx} should respectively read

$$\begin{aligned} D_{zx}^{n+1}(i,j) &= e^{-\sigma_x^D(i)\Delta t} D_{zx}^n(i,j) + \frac{(1-e^{-\sigma_x^D(i)\Delta t})}{\sigma_x^D(i)\Delta x} \\ &\quad \cdot [H_y^{n+0.5}(i+0.5,j) - H_y^{n+0.5}(i-0.5,j)] \end{aligned} \tag{4.1}$$

$$\begin{aligned} B_{zx}^{n+0.5}(i+0.5,j+0.5) &= e^{-\sigma_x^B(i+0.5)\Delta t} B_{zx}^{n-0.5} \\ &\quad \cdot (i+0.5,j+0.5) - \frac{(1-e^{-\sigma_x^B(i+0.5)\Delta t})}{\sigma_x^B(i+0.5)\Delta x} \\ &\quad \cdot [E_y^n(i,j+0.5) - E_y^n(i+1,j+0.5)]. \end{aligned} \tag{4.2}$$

From the above discussion, one can see that once σ^E and σ^H are replaced by σ^D and σ^B the proposed GMIPML can be constructed in a similar manner as that used in Berenger's PML for isotropic media. In addition, $\mathbf{E}$ (or $\mathbf{H}$) is updated through the relation $\mathbf{E} = (1/\varepsilon_0)[\varepsilon]^{-1}\mathbf{D}$ (or $\mathbf{H} = (1/\mu_0)[\mu]^{-1}\mathbf{B}$). This means that inside the GMIPML absorbers $\mathbf{E}$ (or $\mathbf{H}$) can be *simultaneously* absorbed while $\mathbf{D}$ (or $\mathbf{B}$) is *directly* absorbed by the GMIPML.

III. Numerical Validations

To validate and confirm the above theory, hybrid waves propagating in uniaxial anisotropic media consisting of both dielectric ($\varepsilon_1 = 2.0$ and $\varepsilon_2 = 2.2$) and magnetic ($\mu_1 = 2.0$ and $\mu_2 = 2.2$) materials are studied. In particular, if $\theta(\neq 0$ or $\pi/2)$ is the angle between the optical axis and the x direction, then the nonzero elements of the permittivity tensor are

$$\varepsilon_{xx} = \varepsilon_1 \cos^2\theta + \varepsilon_2 \sin^2\theta \tag{5.1}$$

$$\varepsilon_{yy} = \varepsilon_1 \tag{5.2}$$

$$\varepsilon_{zz} = \varepsilon_1 \sin^2\theta + \varepsilon_2 \cos^2\theta \tag{5.3}$$

$$\varepsilon_{xz} = \varepsilon_{zx} = (\varepsilon_2 - \varepsilon_1)\sin\theta\cos\theta \tag{5.4}$$

whereas the nonzero elements of the permeability tensor are fixed as $\mu_{xx} = 2.05, \mu_{yy} = 2.15, \mu_{zz} = 2.0$, and $\mu_{xy} = \mu_{yx} = 0.0866$. For all simulations, the following parameters are used: the computational domain is 20 × 20, the space steps are $\Delta x = \Delta y = 6$ mm, the time step is $\Delta t = 12.5$ ps, and all the results are recorded at 150 time steps. The system is excited by D_z with a smooth compact pulse located at the center point of the domain. The reference FDTD solution is computed with a much larger computational domain.

To demonstrate how the proposed GMIPML absorber works, we first examine how the parameters (such as normal theoretical reflection $R(0)$, power of the conductivity profile n, and number of GMIPML cells N) of the GMIPML absorber affect its absorbing performance. As an example for this examination, the case $\theta = \pi/6$ is considered. Fig. 1(a) shows the normalized local reflections of the E_z field component along a line $(x, 1)$ (i.e., the interface between the GMIPML and the anisotropic medium), caused by the GMIPML with $N = 10$ and different values of $R(0)$ and n. On the other hand, the normalized local reflections, caused by the GMIPML with $N = 15$ and different values of $R(0)$ and n, are given in Fig. 1(b). Noting that when $N = 10$ or 15 the total computational domain is 40 × 40 or 50 × 50. From the results presented in Fig. 1(a) and (b), one can see that although the absorbing performance for $N = 15$ is in general better than that for $N = 10$, a quite good absorbing performance (better than −65 dB) for $N = 10$ can still be obtained while $R(0)$

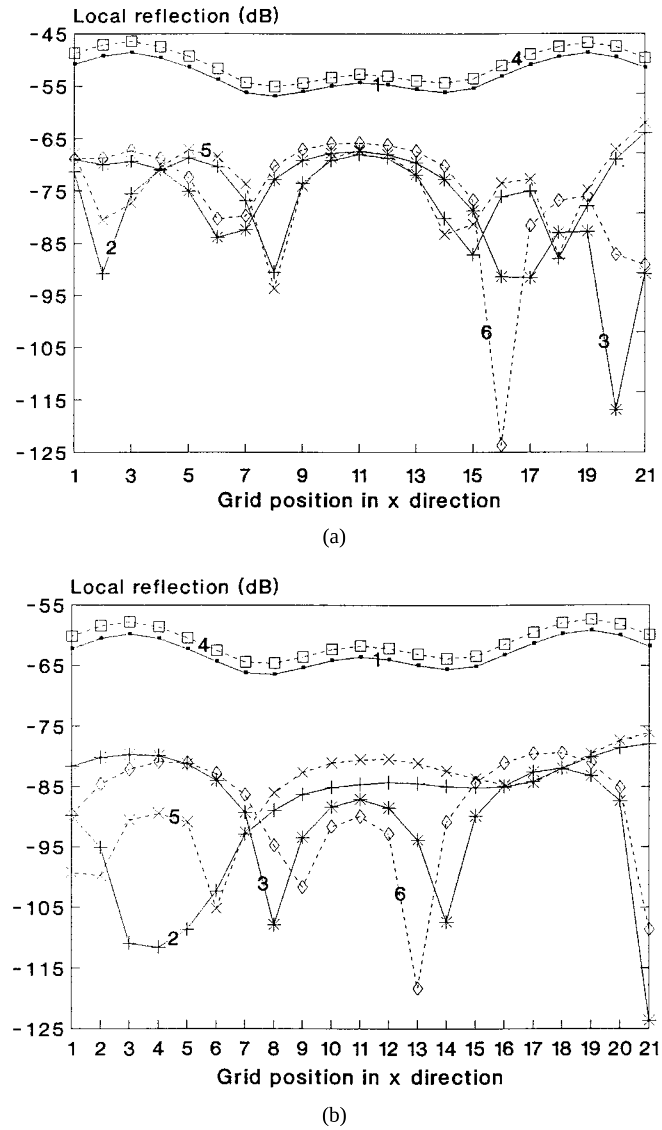

Fig. 1. Effects of different values of N, $R(0)$, and n on the normalized local reflections of E_z field component of the hybrid wave ($\theta = \pi/6$) along line $(x, 1)$, where curves 1–6 are, respectively, for the cases that $(R(0), n)$ = (0.01%, 2); (0.01%, 3); (0.01%, 4); (0.001%, 2); (0.001%, 3); and (0.001%, 4). (a) $N = 10$ for all cases and (b) $N = 15$ for all cases.

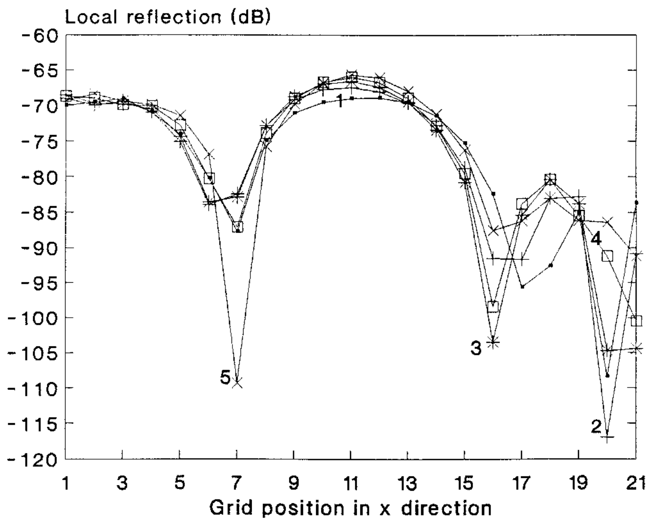

Fig. 2. Normalized local reflections for the E_z field component of waves along line $(x, 1)$, where curves 1–5 are for $\theta = 0, \pi/6, \pi/4, \pi/3,$ and $\pi/2,$ respectively. The parameters of GMIPML used in calculations are $N = 10, R(0) = 0.01\%,$ and $n = 4$.

and n are chosen to be their optimum values. $R(0) = 0.01\%$ and $n = 4$ are the optimum values for the case $N = 10$.

To further evaluate the performance of the GIMPML absorbers, the normalized local reflections for the cases $\theta = 0, \pi/4, \pi/3,$ and $\pi/2$ (noting that the hybrid wave reduces to pure TE or TM wave when $\theta = 0$ or $\pi/2$), caused by the GMIPML absorbers with $N = 10, R(0) = 0.01\%,$ and $n = 4$ are shown in Fig. 2. For comparison, the result of $\theta = \pi/6$ is, again, plotted in Fig. 2. The numerical results shown in Fig. 2 indicate that the proposed GMIPML performs well for all different values of θ. Finally, it should be mentioned here that for simplicity all the fields (including $\mathbf{D}, \mathbf{B}, \mathbf{E},$ and $\mathbf{H}$) were assumed to be zero at the end of the GMIPML absorbers. Certainly, this assumption causes some additional reflections to the computational domain, and the absorbing performance of the present GMIPML absorbers will be further enhanced if any other better assumptions can be found.

IV. Conclusions

In this letter, the GMIPML absorbers that can be applied to general anisotropic materials consisting of both arbitrary permittivity and permeability tensors are proposed. Within these absorbers, $\mathbf{D}$ and $\mathbf{B}$ are directly absorbed whereas $\mathbf{E}$ and $\mathbf{H}$ are simultaneously absorbed. The usefulness and effectiveness of the proposed GMIPML absorbers are confirmed by analyzing hybrid waves in 2-D. Extension of the proposed GMIPML to three-dimensional cases is straightforward. Furthermore, theoretical analysis of the proposed GMIPML absorbers will be reported shortly. Finally, it is anticipated that unsplit formulations (i.e., without splitting the $\mathbf{D}$ and $\mathbf{B}$ fields) can also be derived for the GMIPML absorbers, and the investigation along this line is currently going on.

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